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GROUP DIVISIBLE DESIGNS THROUGH RECODING OF VARIETAL AND LEVEL CODES

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ABSTRACT

Varietal trials and factorial experiments are two main types of experiments which required statistical designs. The required designs for these two types of experiments are different and hence are obtained using different methodologies. Various incomplete block designs can be obtained using these two techniques. In this investigation we have proposed the methods for constructing the singular and semi regular group divisible designs through the recoding of varietal and level codes. Further one method is also developed for the construction of semi regular group divisible designs using Kronecker product of balanced incomplete block designs. The methods are supported with the construction of suitable examples.

KEYWORDS: *Kronecker Products, Varietal Codes, Level Codes, Confounded Interactions and Factorial Experiments*

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1. INTRODUCTION

Bose and Nair (1939) introduced Partially Balanced Incomplete Block Design (PBIBD) which is available for all number of treatments even having a small number of replications. Bose and Shimamoto (1952) classified the PBIBD with two associated class into five categories:

- Group Divisible design
- Simple Partially Balanced Incomplete Block Design
- Triangular type Partially Balanced Incomplete Block Design
- Latin Square type Partially Balanced Incomplete Block Design and
- Cyclic Partially Balanced Incomplete Block Design.

A Group Divisible Design is defined as a design in which the v treatment are divided into m – groups each of n – distinct treatments such that any two treatments belonging to the same group are called first associates and any two treatments belonging to the different groups are second associates. Bose and Conner (1952) classified GD designs into three sub types depending upon the characteristics root of NN’ Matrix as following

(a) Singular, if, $r - \lambda_1 = 0$, (b) Semi – Regular, if $r\,k - v\lambda_2 = 0$, and (c) Regular, if $r\,k - v\,\lambda_2 > 0$.

Several authors like John and Turner (1977), John (1977), Kageyama(1985), Jagdish Prasad et al. (2011)carried out the construction of group divisible designs. Ghosh and Das (1989) discussed the construction of two way group divisible designs. Further Ghosh and Bhimani(1990) carried out different method of construction of group divisible designs. Later Ghosh and Das (1993) carried out the construction of group divisible designs with partial balance for group comparisons. Recently Sharma et al. (2016) discussed the characterization of group divisible designs. Very recently Ghosh and Sinojia (2020) developed the construction of group divisible designs using Hadamard matrix. In this investigation we have developed the construction of group divisible designs by replacing levels of factorial designs with varietal codes of incomplete block designs. In Varietal trials and factorial experiments natural numbers are mostly used for coding for both varietal numbers and level of each factor. Das et al. (1995) provides an alternative method of construction of incomplete block designs for factorial experiments where the level codes of the factor are modified. Here we have modified the concept of Das (1995) to construct singular and semi regular group divisible designs.

2. METHOD OF CONSTRUCTION

Semi regular group divisible designs are constructed through kronecker product of BIB designs and are shown in section 2.1.Semi regular group divisible designs are constructed through levels coded of factorials and are shown in section 2.2, and 3. However singular group divisible designs are constructed through level codes of factorial and are shown in section 4.

2.1 Semi Regular Group Divisible Designs using Kronecker Product of matrices of Balance Incomplete BlockDesigns

Consider a BIB design D_1 with parameters v*, b*, r*, k* and λ*. Next obtain design $D_2$by adding v^* with every elements of design D_1. Using Kronecker product of BIB designs on D_1 and D_2 given by $D = D_1 \otimes D_2$, group divisible designs with parameters $v = 2v^*$, $b = b^{*2}$, $r = r^* b^*$, and $k = 2k^*$is constructed.

Example 2.1: Consider a BIB design with parameters v* = 4 = b*, r* = k* = 3 and λ* = 2. Blocks of the first BIB design D_1 are $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$. Then D_2 is obtained by adding 4 with every elements of design D_1.Therefore blocks of design D_2 are$\begin{bmatrix} 5 & 6 & 7 \\ 5 & 6 & 8 \\ 5 & 7 & 8 \\ 6 & 7 & 8 \end{bmatrix}$. Taking Kronecker product of designs D_1 and D_2 we have finally design D whose blocks are given below

$$\begin{bmatrix} 1 & 2 & 3 & 5 & 6 & 7 \\ 1 & 2 & 3 & 5 & 6 & 8 \\ 1 & 2 & 3 & 5 & 7 & 8 \\ 1 & 2 & 3 & 6 & 7 & 8 \\ 1 & 2 & 4 & 5 & 6 & 7 \\ 1 & 2 & 4 & 5 & 6 & 8 \\ 1 & 2 & 4 & 5 & 7 & 8 \\ 1 & 2 & 4 & 6 & 7 & 8 \\ 1 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 4 & 5 & 6 & 8 \\ 1 & 3 & 4 & 5 & 7 & 8 \\ 1 & 3 & 4 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 5 & 6 & 8 \\ 2 & 3 & 4 & 5 & 7 & 8 \\ 2 & 3 & 4 & 6 & 7 & 8 \end{bmatrix}$$

This is a group divisible design with parameters v= 8, b = 16, r = 12, k = 6 , $\lambda_1 = 8$, $n_1 = 3$, $\lambda_2 = 9$, $n_2 = 4$, n = 4 , m = 2. This group divisible design satisfied r- λ_1> 0 and r k- v λ_2 = 0. Hence the resulting group divisible design is semi regular group divisible design.

Remarks: Clatworthy (1973) has listed the table of partially balanced incomplete block designs with r and k less than and equal to 10. Hence we can claim that this semi regular group divisible design is new as replication size is 12.

2.2 Designs using Blocks of Same PBIB Designs as Level Codes of Factorials

Das et al. (1995) used different set of numbers as level codes of different factors in a factorial. They also used different blocks of a BIB design as level codes of a factor in factorial design to have partially incomplete block design of two associate classes. Instead of BIB design different blocks of a PBIB design as level codes of a factor are used here to construct semi regular group divisible designs.

Theorem 2.1: By replacing s level of s^2 factorial experiment with partially balanced incomplete block design D_1, and incomplete block designs D_2, group divisible design can always be constructed with parameters v = 2v*, b = $b^{*2} = s^2$, r = b* r*, k = 2k*, $\lambda_1 = \lambda_1^*$, $n_1 = n_1^*$, $\lambda_2 = r^{*2}$, $n_2 = \dfrac{b^* r^* (2k^*-1)-n_1^* \lambda_1^*}{r^{*2}}$, n = n_1 + 1 and m = $\dfrac{2v^*}{n_1+1}$.

Proof: Let us consider two factors each at s levels and one PBIB design of two associate classes with parameters v*, b*, r*, k*, λ_1^* and λ_2^* where $b^* = s$. Call this design D_1. Obtain other incomplete block design D_2 by adding v^* with every elements of design D_1. Parameters of this design are identical to design D_1. That is, both the designs are with $b^* = s$ blocks. The varietal codes for the two designs D_1 and D_2 should be different. So that in all 2v* varietal codes can be used. Here we get s^2 combinations of two factors each at s levels and these combinations are written using s different blocks of the two PBIB designs as the level codes of the two factors. Treating the different varietal codes in the two PBIB designs as varieties and the different combination as blocks, a series of incomplete block design is obtained with parameters v = 2v*, b = $b^{*2} = s^2$, r = b* r*, k = 2k*, $\lambda_1 = \lambda_1^*$, $n_1 = n_1^*$, $\lambda_2 = r^{*2}$, $n_2 = \dfrac{b^* r^* (2k^*-1)-n_1^* \lambda_1^*}{r^{*2}}$, n = n_1 + 1 and m = $\dfrac{2v^*}{n_1+1}$.

This series of design becomes semi regular group divisible designs.

Example 2.2: Let us consider a factorial experiment with 2 factors each at 4 levels. As s = 4, the required PBIB design is $D_1$with parameter $v^* = 4 = b^*$, $r^* = k^* = 2$,$\lambda^*_1 = 0$, $n^*_1 = 1$,$\lambda^*_2 = 1$, $n^*_2 = 2$, n = 2, m = 2. The blocks of the PBIB designs are following

$$D_1 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 1 \end{bmatrix} \text{ and } D_2 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 5 \end{bmatrix}$$

Where D_2 is obtained by adding 4(=v) with every elements of design D_1.

The 4^2 = 16 treatment combinations are following:

Table 1

A	B	A	B	A	B	A	B
0	0	1	0	2	0	3	0
0	1	1	1	2	1	3	1
0	2	1	2	2	2	3	2
0	3	1	3	2	3	3	3

Use the four blocks of D_1 as 4 levels of a factor, say, A.Similarly the other four blocks of D_2as levels of another factor, say, B.Replace levels of factor A by four blocks of D_1 and levels of factor B by four blocks of D_2, The following 16 blocks of design D are obtained as

Table 2

1	2	5	6	2	3	5	6	3	4	5	6	4	1	5	6
1	2	6	7	2	3	6	7	3	4	6	7	4	1	6	7
1	2	7	8	2	3	7	8	3	4	5	6	4	1	5	6
1	2	8	5	2	3	8	5	3	4	6	7	4	1	6	7

This is a PBIB design with two associate classes with parameters v = 8, b = 16, r = 8,k = 4, $\lambda_1 = 0$, $n_1 = 1$, $\lambda_2 = 4$, $n_2 = 6$. In fact this PBIB design is a SRGD design as it satisfied $r\text{-}\lambda_1 > 0$ and $r\,k - v\,\lambda_2 = 0$ with n = 2 , m = 4 , $P^1_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix}$ and $P^2_{ij} = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}$.

2.3 Designs using Blocks of Different PBIB Designs as Level Codes of Factorials

In this section we used blocks of two PBIB designs with same parameters but with different varietal codes as level codes of two factors each with the same number of levels. Two different PBIB designs each with a separate set of numbers as varietal codes are considered. The blocks of one design are used as levels of one factor and the blocks of the other design are used as the levels of the other factor. All the combination of the level of these two factors when treated as blocks gives an incomplete block design with unequal number of replication.

3. BLOCK OF PBIB DESIGN AS LEVEL OF THREE FACTORS IN SYMMETRICAL FACTORIAL EXPERIMENTS

Das at al. (1995) discussed the use of fractional factorial experiment with three factors each at s levels. The fraction of factorial experiment is taken in such a way that no main effect and two factors interactions are confounded. However if the fraction is s^2, it is always convenient and easy to get fraction without confounding main effect and two factor interactions.

In this section we used the symmetric factorial experiment with three factors each at s levels. Instead of balanced incomplete block design, partially balanced incomplete block design with two associate classes is used.There are three factors each at s levels, so first develop the s^3 treatment combinations and then construct ans^3 confounding factorial experiment into a block of s^2 block sizes by confounding a suitable interaction so that two factor interaction is saved.Assume this s^2 treatment combination as block of incomplete block design.

Consider a partially balanced incomplete block design with two associate classes whose parameter are v', b', r', k', λ_1' and λ_2' provided b' = s. Let this PBIB design is denoted by D_1. Moreover developed two more incomplete block designs D_2 and D_3 through $v'E_{b'k'} + D_1$ and $v'E_{b'k'} + D_2$ respectively. Parameters of designs D_2 and D_3 are same. That is, all the three design D_1, D_2 and D_3 have same number of blocks, b' = s. However the Varietal codes of all the three designs D_1, D_2 and D_3 are different. Using Theorem 3.1, a group divisible design is obtained.

Theorem 3.1: By replacing s level of s^3confounding factorial experiment cofounded into a block of size s^2 with partially balanced incomplete block design D_1, and incomplete block designs $D_2$and $D_{3,}$ group divisible design can always be constructed with parameters $v=3v'$, $b=(b')^2 = s^2$, $r = b'r'$, $k = 3k'$, $\lambda_1= 0$, $n_1 = 1$, $\lambda_2 = s$, $n_2 = 3v'-2$.

Proof: Let three factors be denoted by A, B and C each at s level where $s \geq 3$.Construct an s^3 confounded factorial experiment by confounding a suitable interaction in to a block of size s^2.Assume each treatment combination of this s^3 confounded factorial experiment as one block. So s^2 treatment combinationsare considered as s^2 blocks. That is, number of blocks $b = s^2$.

Select a partially balanced incomplete block design with b'=s. The parameters of this PBIB design are v', $b' = s$, r', k', λ_1' and λ_2'. Denote this PBIB design by D_1. Again develop two more incomplete block designs using form $v'E_{b'k'} + D_i$. Parameter of these incomplete block designs D_i is identical. All the D_i designs have same number of block, that is, b' = s. However the varietal codes of all the D_i designs are different. We have s^2 treatment combinations from three factors each at s levels, where s levels of each of three factors of s^2 treatment combinations are replaced by s different blocks of 3 incomplete block designs. Since the replication size of the each of the 3 incomplete block design is r_1 and each level of three factors in s^2 confounded factorial design is repeated s times and hence the replication size of the resulting design is $r = sr_1$. Design D_1 has v' treatments, D_2 has another v treatments similarlyD_3 has another v' treatmentsand hence the resulting design has $v'+v'+v' = 3v'$. However s^2 treatment combinations are formed from the combination of three factors and the levels of three factorsare replaced by the blocks of 3 incomplete block designs and hence the block sizes of these incomplete block designs is k'.Therefore the block sizes of the resulting PBIB design is $3k'$.

The varietal code in 3 incomplete block design is different. So for any treatment t_i, there is only one treatment which will not occur together with t_i in any one block of resulting design and hence n_1=1 and λ_1=0. Moreover number of treatment which are second associate of t_1 is n_2 and hence $n_2 = v – 1 - n_1 = 3v' - 2$ as $n_1 = 1$ always. Again each level of three factor are associating with s different varietal cods so $\lambda_2 = s$. Moreover $n_1 = 1$ so n = 2 and $m = (n_2/n) + 1 = [(3v' - 2)/2] + 1$ is an integer provided 3 and v' both should not be odd number. While verification of association matrices and parametric relation of incomplete block design it is observed that the resulting design is a PBIB design with two associate classes. Here n and m comes out as integer and $r – \lambda_1$ is either zero or greater than zero along with $rk - v\lambda_2 =0$ holds true, so PBIB design is a group divisible design with parameters $v = 3v'$, $b = s^2$, $r = sr' = b'r'$, $k' = 3k$, $\lambda_1 = 0$, $n_1 = 1$, $\lambda_2 = s$, and $n_2 =$

$3v^{\prime} - 2$, $p_{ij}^{1} = \begin{pmatrix} 0 & n-2 \\ n-2 & n(n-1) \end{pmatrix}$ and $P_{ij}^{2} = \begin{pmatrix} 0 & n-1 \\ n-1 & n(n-2) \end{pmatrix}$.

Example 3.1: Construct a semi regular group divisible design with parameters v = 12, b = 16, r = 8, k = 6, $\lambda_1 = 0$, $n_1 = 1$, $\lambda_2 = 4$, $n_2 = 10$, n = 2 and m = 6.

Let s = 4, so v = 3 x 4 = 12, b = 4^2 = 16, r = 4x2 = 8, k = 3x2 = 6, $\lambda_1 = 0$, $n_1 = 1$, $\lambda_2 = s$ satisfied the parameters shown in Theorem 3.1. Number of factors are 3 hence total number of combinations s^3=64 are following.

Table 3

A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
B	0	0	0	0	1	1	1	1	α	α	α	α	α^2	α^2	α^2	α^2	0	0	0	0	1
C	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0

Table 4

A	1	1	1	1	1	1	1	1	1	1	1	α	α	α	α	α	α	α	α	α	α
B	1	1	1	α	α	α	α	α^2	α^2	α^2	α^2	0	0	0	0	1	1	1	1	α	α
C	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1

Table 5

A	α	α	α	α	α	α	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2	α^2
B	α	α	α^2	α^2	α^2	α^2	0	0	0	0	1	1	1	1	α	α	α	α	α^2	α^2	α^2	α^2
C	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2	0	1	α	α^2

First construct a 4^3 confounded factorial experiment into a block of size 4^2(= 16) by confounding a suitable interaction ABC. These 16 combinations are following

A : 0 1 0 1 1 1 α 0 α α α α^2 0 α^2 α^2 α^2

B : 0 0 1 1 α α^2 0 α α α^2 1 0 α^2 α^2 1 α

C : 0 1 1 0 α^2 α α α 0 1 α^2 α^2 α^2 0 α 1

Assume these 16 combinations as 16 blocks of the resulting design. Since s = 4 so it required a PBIB design with parameters $v^{\prime} = 4$, $b^{\prime} = 4 = s$, $r^{\prime} = k^{\prime} = 2$, $\lambda_1^{\prime} = 0$, $n_1^{\prime} = 1$, $\lambda_2^{\prime} = 1$ and $n_2^{\prime} = 2$ along with n = 2 and m = 3. The four blocks of this PBIB design D_1 is given below:

$$D1= \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 1 \end{bmatrix}$$

Using $v' E_{b'k'} + D_i$ and i =1,2 we get following two more designs D_2 and D_3 as

$$D_2 = 4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 1 \end{bmatrix}$$

$$D_3 = 4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 10 & 11 \\ 11 & 12 \\ 12 & 1 \end{bmatrix}$$

These three D_1, D_2 and D_3 are incomplete block designs with same parameters with different block structure. Varietal codes of design D_1, D_2 and D_3 are different while $s = b' = 4$, $k' = 2$, $r' = 2$ hold true for all the three incomplete block designs. Suppose four blocks of D_1 as 4 levels of a factor Say, A, similarly the four blocks of D_2 as 4 levels of another factor say B and finally the four blocks of the D_3 as 4 levels of another factor say C.

Replace the 16 treatment combinations of 4^3 confounded factorial experiments by the four varietal codes of A, B and C. Using Theorem 3.1, 16 blocks of the resulting design are given by

Table 6

1	2	1	2	2	2	3	1	3	3	3	4	1	4	4	4
2	3	2	3	3	3	4	2	4	4	4	1	2	1	1	1
5	5	6	6	7	8	5	7	7	8	6	5	8	8	6	7
6	6	7	7	8	5	6	8	8	5	7	6	5	5	7	8
9	10	10	9	12	11	11	11	9	10	12	12	12	9	11	10
10	11	11	10	9	12	12	12	10	11	9	9	9	10	12	11

Since $r - \lambda_1 > 0$ and $rk - v\lambda_2 = 0$, $P_{ij}^1 = \begin{pmatrix} 0 & n-2 \\ n-2 & n(m-1) \end{pmatrix}$ and $P_{ij}^2 = \begin{pmatrix} 0 & n-2 \\ n-1 & n(m-2) \end{pmatrix}$ holds true. Hence the resulting PBIB design is a semi regular group divisible design with parameters $v = 12$, $b = 16$, $r = 8$, $k = 6$, $\lambda_1 = 0$, $n_1 = 1$, $\lambda_2 = 4$, $n_2 = 10$, $m = 2$, $n = 6$, $P_{ij}^1 = \begin{pmatrix} 0 & 0 \\ 0 & 10 \end{pmatrix}$ and $P_{ij}^2 = \begin{pmatrix} 0 & 1 \\ 1 & 8 \end{pmatrix}$.

4. PBIB DESIGN WITH SMALLER NUMBER OF BLOCKS

In this section construction of PBIB design with smaller block size is discussed. Das et. al (1995) obtained PBIB design through symmetrical factorial with $b = s^2$ using blocks of BIB design as levels of three factors. However PBIB design with $b = s$ blocks is constructed here using the same techniques of Das et. al (1995).

Theorem 4.1: By replacing s level of s^3 confounding factorial experiment, cofounded into a block of size s, with blocks of the balanced incomplete block design D_1, and incomplete block designs D_2 and D_3, group divisible design can always be constructed with parameters $v^* = 3v$, $b^* = s = b$, $k^* = 3k$, $r^* = r$, $\lambda_1^* = \lambda + 1$, $n_1 = 2$, $\lambda_2^* = \lambda$, $n_2 = 3(v - 1)$, where v, b, r, k and λdenotes the parameters of a BIB design.

Proof: Assume three factors, say A, B and C each at s level where $s \geq 3$. Construct an s^3 confounded factorial experiment by confounding two suitable interactions in to a block of size s. While confounding two factor interactions precaution should be taken in such a way that each factor containsan equal number of levels in s blocks. Assume each treatment combination of this s confounded factorial experiment as one block. Thus number of blocks b = s.Select a balanced incomplete block design with b =s, whose parameters are v, b = s, r, k, and λ. Denote this BIB design by D_1. Further D_x more designs are obtained using $(x - 1)vE_{bk} + D_1$, where x = 2, 3. Parameters of these incomplete block designs $D_i$are identical. All the D_i designs have same number of block, that is, b = s. However the varietal codes of all the D_i designs are different. Here design D_1 has v treatments, D_2 has another v treatment similarly D_3 has also another v treatments so number of treatment for resulting design is 3v. Replacing s levels of factors A, B, C by level codes of s blocks of D_i (i = 1, 2, 3), PBIB design with parameters $v^* = 3v$, $b^* = s = b$, $k^* = 3k$, $r^* = r$, $\lambda_1^* = \lambda + 1$, $n_1 = 2$, $\lambda_2^* = \lambda$, $n_2 = 3(v - 1)$ is obtained.

Example 4.1: ConstructSingular Group Divisible design with parameters $v^* = 9$, $b^* = 3$, $r^* = 2$, $k^* = 6$, $\lambda_1^* = 2$, $n_1^* = 2$, $\lambda_2^* = 1$, $n_2^* = 6$, $n = 3$, $m = 3$.

Assuming s = 3,construct a 3^3 symmetrical factorial experiment in to a block of size 3 by confounding suitable two factor interaction AB^2 and three factor interactions ABC^2. This design is given below:

A	B	C
0	0	0
1	1	2
2	2	1

Note that number of level of each factor A, B and C is one. That is,levels 0, 1 and 2 occur only one time for each factor. Construct one BIB designs with parameters v = b = 3, r = k = 2 and λ = 1. The blocks of this BIB design, D_1 are

1 2

2 3

3 1

Designs D_2 and $D_3$are obtained by adding 3 and 6 with each element of design D_1 respectively. The blocks of designs D_2 and D_3 are following

$$D_2 = \begin{matrix} 4 & 5 \\ 5 & 6 \\ 6 & 4 \end{matrix} \qquad D_3 = \begin{matrix} 7 & 8 \\ 8 & 9 \\ 9 & 7 \end{matrix}$$

Replacing the three levels of A, B and C by the three blocks of D_i (i = 1, 2, 3) respectively we get the following design :

1	2	4	5	7	8
2	3	5	6	9	7
3	1	6	4	8	9

This design is Singular Group Divisible design with parameters $v^* = 9$, $b^* = 3$, $r^* = 2$, $k^* = 6$, $\lambda_1^* = 2$, $n_1^* = 2$, $\lambda_2^* = 1$, $n_2^* = 6$, n = 3 , m = 3.

This design is not a new design as it is reported as S - 21 in Clatworthy (1973).

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